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AN EFFICIENT PRESENTATION FOR THE GROUP SL (2,121)

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Abstract

In this paper we give a 2-generator 2-relation presentation for the group SL(2,121). This proves that SL(2,121), the covering group of PSL(2,121), is efficient.

Introduction

A finite group is said to be *efficient* if it has a presentation on d generators and d+rank (M (G)) relations where M(G) is the Schur multiplier of G, [7,6]. The group H is a *stem extension* of G if there is an A≤H with H/A ≅ G and A≤Z(H) ∩H' . If A ≅ M (G) then H is a covering group of G, see [7]. It is proved in [7] that any stem extension of a group H is a homomorphic image of some covering group of G.

If G is a finite simple group then G has a unique covering group $\widehat{G}$ and M($\widehat{G}$) =1 , see [7]. Hence, to prove that $\widehat{G}$ is efficient, we need to find a presentation for $\widehat{G}$ with an equal number of generators and relations.

The problem of finding such presentations for the covering group of finite simple groups G, $|G| < 10^6$, has been surveyed in [3]. Among the simple groups PSL $(2, p^n)$, p odd prime, of order less than 10^6 only the efficiency of SL(2,121), the covering group of PSL(2,121), see [6], was unknown. Here we give a 2-generator 2-relation presentation for SL(2,121) =P$\widehat{S}$L (2,121). This proves that SL(2,121) is efficient.

A 2- Generator 2- Relation Presentation for SL(2,121). In obtaining a 2- generator 2- relation presentation for SL(2,121) we first attempt to find a 2-generator 3- relation presentation for PSL(2,121) using the method described in [5]. Then by combining two relations of the presentation obtained we try to arrive at a presentation for SL(2,121).

We start with the following permutation generators a and b for PSL(2,121) given on the CAYLEY file described in [2].

a= (1,3) (2,18) (4,43) (5,75) (6,63) (7,61) (8,48) (9,90) (10,49) (11,79) (12,97) (13,14) (15,68) (16,84) (17,56) (19,32) (20,30) (21,94) (22,58) (23,118) (24,64) (25,50) (26,59) (27,35) (28,31) (29,33) (34,67) (36,114) (37,62) (38,76) (39,44) (40,46) (41,47) (51,116) (52,120) (53,54) (55,112) (57,80) (60,122) (65,82) (66,111) (69,72) (70,83) (71,92) (73,104) (74,99) (77,86) (78,96) (81,107) (85,95) (87,102) (88,106) (89,93) (91,109) (98,108) (100,101) (103,119) (105,110) (113,115) (117,121) .

b= (1,89,5) (2,48,72) (3,120,21) (4,62,101) (6,51,11) (7,108,113) (8,115,103) (9,19,77) (10,26,24) (12,49,55) (13,80,42) (15,86,52) (16,104,57) (17,95,27) (18,50,78) (20,58,98) (22,90,97) (23,44,87) (25,67,56) (28,53,82) (29,47,119) (30,66,109) (31,93,79) (32,39,64) (33,60,68) (34,118,106) (35,38,100) (36,112,88) (37,122,92) (40,102,94) (41,76,61) (43,91,63) (45,111,117) (46,107,70) (54,73,99) (59,116,65) (69,114,74) (71,75,83) (81,84,110) (85,105,96).

We note that a presentation for PSL(2,121) on the

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generators a and b is given in [2] as follows:
$G = \langle a,b \mid a^2 = b^3 = (ab)^{61} = (ababab^{-1})^6 = ((ab)^6 ab^{-1} (ab)^3 ab^{-1})^2 = ((ab)^7 ab^{-1} (ab)^2 (ab^{-1})^2)^2 = 1 \rangle$.

This presentation is not a 'small presentation and fails to be reduced to a 2-generator 3-relation presentation. So that it is reasonable to start with a presentation having fewer relations than those of G as much as possible. In an attempt to obtain such a presentation we find that $x=a$ and $y= bab^{-1}(ab)^4(ab^{-1})^3abab^{-1}$ generate PSL(2,121) using the CAYLEY system [4]. Then the computer program PERM as described in [1] was used to construct the following relation:

$x^2 = y^3 = 1$

$(xy)^5(xy^{-1})^2 xyxy^{-1}(xy)^2((xy^{-1})^2xy)^3 (xyxy^{-1})^2 xy^{-1} = 1$.

A difficult coset enumeration using a computer implementation of the Todd-Coxeter algorithm showed that we have sufficient relations to define the group PSL(2,121). This was run on a SUN computer at the University of St. Andrews.

Now taking

$H = \langle x,y \mid x^2y^3 = 1, (xy)^5(xy^2)^2xyxy^2(xy)^2((xy^2)^2xy)^3 (xyxy^2)^2xy^2 = 1 \rangle$,

we observe that H is a stem extension of SL(2,121). Therefore, $|H| \leq 2|PSL(2,121)|$ since $M(PSL(2,121)) = C_2$, the cyclic group of order 2, see [6]. Hence $H \cong SL(2,121)$ or $H \cong PSL(2,121)$. However, the latter is impossible because PSL(2,121) requires at least 3 relations on 2 generators.

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